

اسم و ترمیم منظر نظریة الاحتمالات  
 حساب ریاضیات، فاع (۱۰۱۵ - ۱۰۱۶)

ص: (۵۰ درم)

۱) لدينا  $f(x) = 1, x \in [0, 1]$  عندئذ:

۱)  $g_y(y) = f_y(x) \left| \frac{dx}{dy} \right|_{x=y} = e^{-y} = e^{-y}, y > 0$  ③  
 $\lambda = 1$  صر

2)  $F_y(y) = 1 - e^{-y}, y > 0$  ③

3)  $M_z(t) = M_{Z_{Y_1}}(t) = (M_Y(t))^n = (1-t)^n$  ⑥  
 ⑥  $M_Y(t) = (1-t)^n$  صر

3)  $P(Y > 1) = 1 - F_Y(1) = 1 - (1 - e^{-1}) = e^{-1} = \frac{1}{e}$  ③

5)  $E(3Z) = 3E Z = 3 \frac{1}{\lambda} = 3n$  ②  
 $V(3Z) = 9V(Z) = 9n$  ②

6)  $M_W(t) = E e^{t \ln\left(\frac{1-x}{x}\right)} = E (1-x)^t x^{-t} = \int_0^1 x^{t-1} (1-x)^{t-1} dx = \frac{1}{2} (1-t, t)$  ②

7)  $EY_1 Y_2 = EY_1 EY_2 = (1)(1) = 1$  ③,  $M_{Y_1, Y_2}(t_1, t_2) = (1-t_1)(1-t_2)$  ③

$p(y_1, y_2) = p(y_1, y_2) = 1$  ①

2)  $P(y_1, y_2) = F_Y(y_1) F_Y(y_2) = (1 - e^{-y_1})(1 - e^{-y_2}), y_1 > 0, y_2 > 0$  ③

3)  $P(Y_1 > 1, Y_2 > 1) = (1 - F_Y(1))(1 - F_Y(1)) = e^{-1} \cdot e^{-1} = e^{-2} = \frac{1}{e^2}$  ③

4)  $g_u(u) = 2[1 - F_Y(u)] f_Y(u) = 2e^{-2u}$  ④  
 $u > 0$

5)  $g(z_1, z_2) = p(y_1, y_2) \left| \frac{y_1}{z_1} \frac{y_2}{z_2} \right|_{y_1=z_1, y_2=z_2} = \frac{1}{z_1 z_2} e^{-z_1 - z_2}$  ④  
 $z_1 > 0, z_2 > 0$

1)  $F(x/y) = 1 - e^{-\frac{x}{2y}}, x > 0, y > 0$  ③

2)  $P(X > 4/Y=2) = 1 - F(4/2) = 1 - (1 - e^{-1}) = e^{-1}$  ③

3)  $M_{X/Y}(t) = (1 - 2yt)^{-1} = (1 - 2t)^{-1}, E(X/Y=1) = 2y|_{y=1} = 2$  ②

1)  $M_X(t) = e^{-2(1-e^t)}$  ④,  $K(t) = -2(1-e^t), M(\ln t) = e^{-2(1-t)}$  ④  
 $M_X(t) = e^{-2t} M_X(t) = e^{-2t} e^{-2(1-e^t)}$  ④

$$2) P(X+Y=n) = \sum_{i=0}^n P_X(i) P_Y(n-i) \stackrel{(2)}{=} e^{-4} \sum_{i=0}^n \frac{4^i}{i!} \frac{4^{n-i}}{(n-i)!} \stackrel{(2)}{=} e^{-4} \frac{4^n}{n!}, n=0,1,\dots$$

$$P(X=K/X+Y=n) = \frac{P(X=K)P(Y=n-K)}{e^{-4} \frac{4^n}{n!}} \stackrel{(2)}{=} \frac{e^{-4} \frac{4^K}{K!} e^{-4} \frac{4^{n-K}}{(n-K)!}}{e^{-4} \frac{4^n}{n!}} \stackrel{(2)}{=} \frac{n!}{K!(n-K)!} \left(\frac{1}{2}\right)^K \left(\frac{1}{2}\right)^{n-K} \stackrel{(2)}{=} \binom{n}{K} \left(\frac{1}{2}\right)^n, K=0,1,\dots,n$$

$$\varphi_X(t) = e^{-2|t|}, t \in \mathbb{R}$$

در اینجا

$$1) f_X(t) = \frac{2}{\pi} \frac{1}{4+t^2}, t \in \mathbb{R} \quad (4) \quad \text{توزیع کاهانه و دایره ای}$$

$$2) F_X(t) = \frac{1}{2} + \frac{1}{\pi} \arctan \frac{t}{2}, t \in \mathbb{R} \quad (4)$$

$$3) \varphi_{\frac{X}{n}}(t) = \left( \varphi_X\left(\frac{t}{n}\right) \right)^n = \left( e^{-2|t/n|} \right)^n = e^{-2|t|} \quad (4)$$

اگر  $X_1, X_2$  همبسته باشند، پس  $X_1$  و  $X_2$  همبسته هستند.

$$4) F_{(X_1, X_2)}(x_1, x_2) = F_{X_1}(x_1) F_{X_2}(x_2) = \left( \frac{1}{2} + \frac{1}{\pi} \arctan \frac{x_1}{2} \right) \left( \frac{1}{2} + \frac{1}{\pi} \arctan \frac{x_2}{2} \right), x_1, x_2 \in \mathbb{R} \quad (2)$$

$$5) P(X_1 < 2, X_2 < 2) = F_{X_1}(2) F_{X_2}(2) = \frac{3}{4} \cdot \frac{3}{4} = \frac{9}{16} \quad (2)$$